A Numerical Method for the Ferromagnetic Granules Utilizing DEM and MoM

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The ferromagnetic granules / powders under a magnetic field show a complex behavior. The behavior generated by an interaction between ferromagnetic granules / powders and an external magnetic field. These phenomena are applied to the forming process of bond magnets, ferrofluid, MR fluid and magnetic brash in photography. However, it is difficult that modeling of ferromagnetic granules / powders. The purpose in this report is proposing a numerical simulation method for ferromagnetic granules. In this paper, the coupling method employing the Discrete Element Method and Method of Moment is described. Additionally, the proposal method is applied to the numerical model of ferromagnetic granules under a static magnetic field.

***Index Terms*—Method of Moments, Numerical Simulation, Granular Computing**

I. Introduction

THE ferromagnetic granules behavior is applied to many territory that include a forming process of bond magnet, ferrofluid and MR fluid, magnetic brash image development method. However, this behavior shows a complex phenomenon that include an interaction between the granules and external magnetic field. Therefore, it is difficult to control these behavior.

The numerical calculation techniques are employed to CAE for these design process. The Finite Element Method is employed as general method for the CAE technology [1]-[2]. However, these methods require the mesh. Additionally, the remeshing process is required whenever structural boundary of object transforms. Besides, the analysis possibly breaks down during the remeshing process.

In this paper, the numerical calculation method for ferromagnetic granules under a static magnetic field is proposed. In this study, the Discrete Element Method [3,4] and Method of Moments [5] are combined in weakly coupling. The coupling method and algorism are described. Finally, the effectiveness and usefulness are discussed through the calculation result of ferromagnetic granules under a static magnetic field.

II. Employed Algorisms

In this study, the numerical calculation method for multi-physics phenomena is proposed. The proposal method employs the Discrete Element Method (DEM) and Method of Moments (MoM).

The granules behavior is calculated by DEM. In DEM, collision between granules is modeled by Voigt model. Also, the motion equation of translation and rotation are treated in DEM.

$$m\frac{d^2\boldsymbol{x}}{dt^2}+c\frac{d\boldsymbol{x}}{dt}+k\boldsymbol{x}=\boldsymbol{F} \tag{1}$$

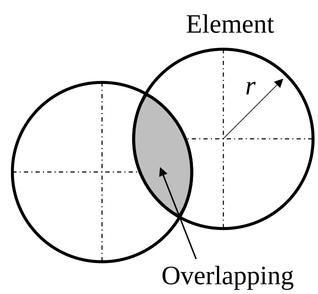

Fig. 1 Positional relationship of elements.

$$I\frac{d^2\boldsymbol{p}}{dt^2}+cr^2\frac{d\boldsymbol{p}}{dt}+kr^2\boldsymbol{p}=\boldsymbol{T}. \tag{2}$$

The m is a mass, $\boldsymbol{x}$ is a displacement of element, c is a damping coefficient, k is a spring constant, $\boldsymbol{F}$ is external force, I is an inertia, $\boldsymbol{p}$ is a rotational displacement, r is a radius of element and $\boldsymbol{T}$ is torque. The collision is modeled by penetration of the elements during collision. The contact force is calculated by an overlapping of the elements (Fig. 1). Therefore, the overlapping of the elements is tolerated in DEM. Additionally, linear spring coefficient can be employed if the overlapping is sufficiently small.

On other hands, Static magnetic field distribution is calculated by MoM. In MoM, the integral equation about magnetization M is treated [6].

$$\frac{\chi_i}{4\pi}\sum_j\left[\nabla\int_V \boldsymbol{M}\cdot\nabla\left(\frac{1}{r_{ij}}\right)dV\right]+\boldsymbol{M}_i=\chi_i\mu_0\boldsymbol{H}_i \tag{3}$$

where, χ_i is the magnetic susceptibility of element i, $\boldsymbol{M}_i$ is the magnetization vector on element i, r_{ij} is the distance between ferromagnetic element i and j, V is the volume of element i, μ_0 is the permeability on free space and $\boldsymbol{H}_i$ is the external magnetic field strength. The equation just means the interactions between a ferromagnetic material element i and others. Eq. (3) is formulated about all elements. As the result, obtaining magnetizations of all elements results in solving simultaneous linear equation of magnetization as follows:

$$\left\{\begin{bmatrix} C_{11} & \cdots & C_{1n} \\ \vdots & \ddots & \vdots \\ C_{n1} & \cdots & C_{nn} \end{bmatrix} + I \right\} \begin{bmatrix} \boldsymbol{M}_1^T \\ \vdots \\ \boldsymbol{M}_n^T \end{bmatrix} = \begin{bmatrix} \chi_1 \mu_0 \boldsymbol{H}_1^T \\ \vdots \\ \chi_1 \mu_0 \boldsymbol{H}_n^T \end{bmatrix}. \tag{4}$$

Here, I is the $3n$ order identity matrix, C is the 3 order brock matrix of coefficient matrix. The magnetic force and torque per unit volume on certain element presented by

$$\boldsymbol{f} = \nabla(\boldsymbol{M} \cdot \boldsymbol{H}_e) + \frac{1}{2}\nabla(\boldsymbol{M} \cdot \boldsymbol{H}') \tag{5}$$

$$\boldsymbol{\tau} = \boldsymbol{M} \times (\boldsymbol{H}_e + \boldsymbol{H}') \cdot \tag{6}$$

Here, $\boldsymbol{H}_e$ is the external magnetic field by source. $\boldsymbol{H}$' is the external magnetic field by other elements. These assigned to the external force and torque term of Eqs. (1), (2). After that, velocity and position of each elements are updated.

III. Calculation Result

The proposal method applied to ferromagnetic granules under a static magnetic field. The numerical model is shown in Fig. 2. The analysis conditions are shown in TABLE I. In this case, the damping coefficient in DEM is presented by

$$c = 2\gamma\sqrt{mk} \tag{7}$$

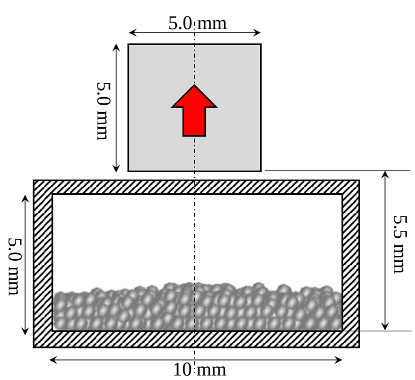

Fig. 2 Schematic diagram of numerical model.

TABLE I
ANALYSIS CONDITIONS

Parameter	Value
Number of elements	1100
Permanent magnet magnetization [T]	0.2
Element diameter [mm]	0.5
Spring coefficient [N/m]	800
Density[kg/m^3]	7874
Coefficient of restitution	0.8
Dynamic friction coefficient	0.8
Relative permeability	3000
Time interval [s]	1e-06
Number of steps	50000
CPU time [h]	18.12

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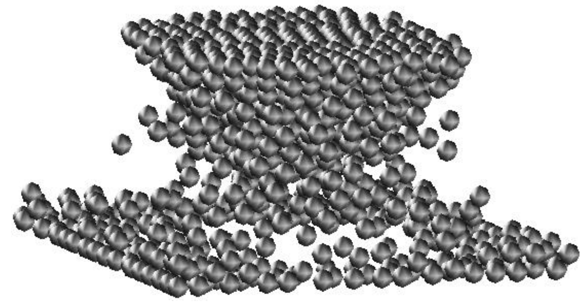

Fig. 3 Analyzed result (50 ms)

$$\gamma = \frac{\alpha}{\sqrt{1-\alpha^2}} \tag{8}$$

$$\alpha = -\frac{1}{\pi}\ln e \cdot \tag{9}$$

Where, e is the coefficient of restitution. The analyzed result is shown in Fig. 3. In Fig. 3, the column of ferromagnetic granules is generated under the permanent magnet. The ferromagnetic elements are attracted to the permanent magnet, these elements behave as a flux pass. As the result, the column of ferromagnetic granules is formed. Also, the experimental result will be shown in full paper.

IV. Conclusion

In this paper, the calculation technique utilizing the Discrete Element Method and Method of Moments is proposed. The method does not require the mesh. Also, the compatibility of DEM and MoM is high. Therefore, the same numerical model is used through the calculation process, the coordinate and physical value transformations does not required. Besides, these can be coupled seamlessly.

In the calculated result, the ferromagnetic granules swarm had been attracted to the permanent magnet, the column of ferromagnetic granules was generated. It is possible to simulate a ferromagnetic granules behavior under a static magnetic field by the proposal method.

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